ALLOWANCE FOR THE EFFECT OF PRESSURE IN THE THEORY OF SECOND-ORDER PHASE TRANSITIONS (WITH APPLICATION TO THE CASE OF SUPERCONDUCTIVITY)

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The effect of pressure on second-order phase transitions is discussed. Specifically, the superconducting transition is considered.

In the theory of second-order phase transitions one usually considers only the temperature dependence of a number of quantities near the transition point.[1] Nevertheless the pressure dependence, for example the dependence of the characteristic transition parameters η_i, is also of interest. The components of the spontaneous electrical polarization or magnetization vectors are usually taken as η_i in ferromagnetics and ferroelectrics. In superconductors $\eta^2 \sim |\Psi|^2$, where Ψ is the effective wave function of superconducting electrons,[2] and in pure isotropic superconductors $\Psi \sim \Delta$, where Δ is the gap in the energy spectrum.[3] Within the framework of the approximate theory of phase transitions[1] allowance for pressure is quite obvious. Nevertheless we shall consider here this problem with reference to superconductors for the following reasons. In the region of the superconducting transition at $T \neq 0$ the theory[1,2] is practically exact.[4] Moreover, in the case of superconductivity, using the microscopic BCS theory,[5] it is possible to obtain some additional information for the model considered. Finally, it is in the case of superconductors that the pressure dependence of various quantities can be determined easily by experiment and in particular, as follows from the work of Brandt and N. I. Ginzburg,[6] one can hope to study in sufficient detail the dependence $T_c(p_c)$.

Bearing in mind the application to superconductivity we shall from the outset denote the characteristic parameter η by Δ and we shall write the well-known expansion of the thermodynamic potential in the form (the quantity Δ is assumed to be real, otherwise Δ^2 should be replaced by $|\Delta|^2$)

$$\Phi(p, T) = \Phi_0(p, T) + \alpha(p, T)\Delta^2 + \tfrac{1}{2}\beta(p, T)\Delta^4 + \ldots \quad (1)$$

Along the line of the second-order phase transition $\alpha(p_c, T_c) = 0$. At a fixed pressure near the transition point we can assume

$$\alpha(p_c, T) = \left(\frac{\partial\alpha}{\partial T}\right)_{p_c, T_c}(T - T_c),$$

as is usual.[1-4] Similarly at a fixed temperature we can assume

$$\alpha(p, T_c) = (\partial\alpha/\partial p)_{p_c, T_c}(p - p_c).$$

Moreover, at equilibrium in the ordered phase $\Delta^2 = -\alpha/\beta$. Hence we obtain the dependence

$$\Delta(p, T_c(p_c)) = \left[\frac{(\partial\alpha/\partial p)_{p_c, T_K}}{\beta(p_c, T_c)}(p_c - p)\right]^{1/2}. \quad (2)$$

Here of course $T_c(p_c)$ denotes the temperature at which measurements are carried out at the pressure p (the temperature $T_c(p_c)$ is the critical temperature only for the pressure p_c). The dependence $\Delta(p)$ can be measured by various methods. At the same time it follows from the expansion (1) that $H_{cb}^2/8\pi = \alpha^2/2\beta$ (cf.[2]), i.e., the critical magnetic field for bulk samples $H_{cb}(p, T_c(p_c)) \propto (p_c - p)$.

For the λ-transition in liquid helium ρ_s (cf.[7]) is taken as Δ^2 in Eq. (1), and therefore $\rho_s(p, T_\lambda(p_\lambda)) \propto (p_\lambda - p)$, if one ignores the possible need of allowing in Eq. (1) for a term of higher order [i.e., a term $(1/6)\gamma\rho_s^3$]. However, in this case the region of applicability of the expansion of type (1) is not sufficiently clear. This difficulty makes a study of the dependence $\rho_s(p, T)$ near the λ-point even more interesting.

The shape of the curve $p_c(T_c)$ for which $\alpha(p_c, T_c) = 0$ is in general not known. Moreover, for second-order transitions we cannot even state that necessarily $dp_c/dT_c \to 0$ as $T \to 0$. In fact, for a second-order transition

$$\delta\left(\frac{\partial V}{\partial T}\right)_p = -\frac{dp_c}{dT_c}\,\delta\left(\frac{\partial V}{\partial p}\right)_T$$

(cf. [1]) and, on the other hand, $(\partial V/\partial T)_p \to 0$ as $T \to 0$. Therefore $\partial p_c/\partial T_c = 0$ when $T_c \to 0$ only when the compressibility discontinuity $\delta(\partial V/\partial p)_T$ at the transition is not equal to zero.

In the BCS microtheory of superconductivity the gap is

$$\Delta(p_c, 0) = 3.52\,KT_c(p_c) = 4\hbar\omega_c \exp[-1/N(0)V]$$
$$\equiv f(p_c)\exp[-1/\varphi(p_c)],$$

but an actual analysis of the dependence of T_c on p_c has not yet, to our knowledge, been carried out. One may think that in the absence of any phase transition of other type the gap closes up [i.e., $\Delta(p_{c0}, 0) = 0$, $T_c(p_{c0}) = 0$] at some finite pressure p_{c0}. If in the pressure region close to p_{c0} we use the formula given above for $\Delta(p_c, 0)$ (cf. also [8]), then $\varphi(p_{c0}) = 0$ and probably $\varphi(p_c) \propto (p_{c0} - p_c)$. It is obvious that $dT_c/dp_c = 0$ at $T_c = 0$ and the gap $\Delta(p_c, 0)$ depends exponentially on the difference $(p_{c0} - p)$. Thus for superconductors at $T = 0$ and in several other cases, [9] we do not have a second-order transition. However, at any temperature $T \neq 0$ the results of the microtheory represent the model based on the expansion (1). In fact, according to the BCS theory, near the critical temperature we can write

$$\Delta(p, T) = 6.12\,kT_c(p)\,[1 - T/T_c(p)]^{1/2}.$$

To fix the temperature T we shall denote it by $T_c(p_c)$. Then we obtain immediately, by expanding the radicand as a series in $(p_c - p)$, a formula of type (2).

At the same time it is understood that the phenomenological approach is more widely applicable in the sense that it is not connected with the use of any special model. In particular, in the anisotropic case, when the gap width depends on direction, it is necessary to replace $|\Delta|$ by $|\Psi|$ in the expansion (1), but as before $|\Psi| \propto \sqrt{p_c - p}$ and $H_{cb} \propto (p_c - p)$.

It is also easy to use the expansion (1) assuming that α and β depend on the stress tensor σ_{ij} or the deformation tensor u_{ij}. Within the framework of the theory of Landau and the present author and its generalization to the anisotropic case, [10] it is natural also to consider the dependence on pressure (or on σ_{ij}) of such quantities as the depth of penetration of the field, surface energy at the boundary between the normal and superconducting phases, etc.

It is worth noting that in the case of ferroelectrics and ferromagnetics the effect of pressure on a second-order transition has already been considered earlier,[11,12] although in a form somewhat different from that used above.

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